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STABILITY OF NUMERICAL INTEGRATION TECHNIQUES FOR TRANSIENT ROTOR DYNAMICS

by Albert F. Kascak

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SUMMARY

A nonlinear finite element model of a rotor bearing system was linearized at some instant of time. The stiffness and damping coefficients were assumed to be constant over a short time period at the instantaneous values. A numerical stability analysis was applied to the linearized model to determine the stability limits of the forward, backward, and centered Euler; the fourth-order Runge-Kutta; the Milne; and the Adams methods of numerical integration of the set of equations of motion.

The stability analysis obtained the error growth rate (the ratio of the amplitudes of the errors for two succeeding time steps) as a function of a nondimensional time step (the product of the time step and the natural frequency) and of the damping ratio for each mode. The absolute error growth rate was independent of the actual transient; the relative error growth rate was not. The relative error growth rate was normalized by the transient solution of the perfectly balanced rotor.

For any mode with a nonnegative damping ratio, the absolute error growth rate must be less than or equal to 1. (If not, the numbers used in the calculation will become too large for the computer.) If a mode is to be calculated accurately, the relative error growth rate must also be less than or equal to 1. The lower frequency modes are the only modes that are physically meaningful, although all the modes are inherently present in the calculation of any transient.

Usually, the highest frequency mode possible in the finite element model determines the threshold of numerical stability. Therefore, the number of mass elements in the finite element model should be minimized. Increasing the damping of a mode can cause it to become numerically unstable. An example considered is that of a uniform shaft on rigid bearings, with 10 mass elements, and operating at approximately the first critical speed. The maximum time step for the Runge-Kutta, Milne, and Adams methods is that which corresponds to approximately 1 degree of shaft movement. To calculate the shaft motion to several revolutions would thus require thousands of time steps.

INTRODUCTION

The simulation of transient rotor dynamics by a computer is done by one of two methods. The first is the modal method. This method is applicable to linear problems. It consists of integrating the equations of motion for each mode separately and then summing them to get the final result. The second method is the finite element method. This method is applicable to nonlinear problems. It consists of modeling the rotor dynamics system by a finite number of elements and then integrating the equations of motion for all of these elements simultaneously.

References 1 and 2 use the finite element approach. Reference 1 is the more general of the two codes. Besides simulating more features of the rotor dynamics system, it includes several choices of numerical method for integrating the equations of motion. Reference 1 concluded that the Adams-Moulton predictor-corrector integration technique gave the best combination of computational speed and accuracy. Application of this code to other problems, such as that of reference 3, has been frustrated by numerical stability problems.

Reference 2 is the more restrictive of the two codes. It uses a modified (centered) Euler technique to integrate the equations of motion. This code has been applied successfully to the problem of reference 3. The question arises as to why the code of reference 2 worked and that of reference 1 did not. A partial answer to this question can be found in the numerical stability of the various integration techniques.

This report discusses the numerical stability of some typical integration techniques as applied to transient rotor dynamics. The set of equations simulating the transient rotor dynamics would be classified mathematically as a stiff set of equations. (A set of equations is stiff if the ratio of the largest to the smallest eigenvalue is large, i.e., more than 100 to 1.) Reference 4 discusses numerical stability and proposes a numerical integration technique, Gear's method, for a general set of stiff equations. This method was applied to the code of reference 1 but required too much computing time.

The numerical stability analyses of references 4 and 5 were applied to some typical numerical methods presented in reference 6. The present study examined the numerical stability of the forward, backward, and centered Euler methods; the fourth-order Runge-Kutta method; the Milne method; and the Adams method as applied to a transient rotor dynamic simulation.

The finite element method divides the shaft into a number of axial elements. The acceleration of the elements is related to the sum of the forces on the elements. These forces are basically the elastic force, the drag force, and the inertial unbalance force. The elastic force is related to the shaft stiffness and therefore to the displacements of the elements. The drag force is related to the velocity of the elements. The unbalance

force is independent of the displacement and velocity of the elements and is therefore the "forcing function."

The numerical integration technique replaces the time-dependent set of differential equations of motion by a set of difference equations. When the time step is small, the solution to the difference equations approximates the solution to the differential equations. When the time step in the difference equations exceeds a critical value, the solution no longer approximates the solution of the differential equations of motion. The relative error between the two solutions increases with time. The numerical integration technique is unstable for time steps greater than the critical value since a small error will increase to a large error.

This critical time step is not generally known since the solution to the differential equations of motion is not known. If the problem is linearized in time (i.e., the stiffness and damping of the elements are assumed to be constant at the instantaneous values), a modal analysis can be applied to the instantaneous mode shapes and frequencies for both the differential and difference equations. The relative error in the amplitude of each mode must not increase with time. The stability analysis applies for the linearized problem; therefore, an extension must be made between the linearized and nonlinear problems (i.e., if the linearized problem is unstable, the nonlinear problem is also unstable).

Since the problem is linearized, the absolute error can be calculated independently from the actual simulation. The error must satisfy the same set of homogeneous equations, but it has a different forcing function (the rate at which the computer generates errors). If the computer rounds the last significant figure (rather than truncating), the forcing function, on the average, is zero. The error analysis then reduces to an initial-value problem that can be solved analytically for each numerical integration method.

In order to determine the relative error, the absolute error must be normalized by the solution to the linearized nonhomogeneous differential equations of motion. This solution depends on the specific transient. In order to generalize the results, the most conservative transient is used to normalize the absolute error; that is the solution to the homogeneous differential equations of motion. The solution to the homogeneous equation of motion (i.e., the shaft in perfect balance) will yield the smallest vibrations and therefore the largest relative error.

ANALYSIS

This analysis assumes a model of a rotor bearing system that is linearized at some instant of time and neglects both torsional windup and gyroscopic moments. Figure 1 shows a model of the shaft with n finite elements. The complex number representation

of the radial displacements of the shaft centerline from the axis of rotation for the i^{th} finite element is r. (Symbols are defined in the appendix.)

If R is a column matrix of the displacements of the shaft centerline, if the column matrix V is the time derivative of R, and if the column matrix A is the time derivative of V, the second-order matrix equation of motion for the rotor bearing system can be written as

$$MA + CV + KR = F$$

where M is a diagonal matrix of the masses, C is a square matrix of the damping coefficients, K is a square matrix of the stiffness coefficients, and F is a column matrix of the forces. The following partitioned matrices can be defined:

$$Z = \begin{bmatrix} R \\ - - - \\ V \end{bmatrix}$$

$$U = \begin{bmatrix} V \\ - - - \\ A \end{bmatrix}$$

$$P = -M^{-1}\left[\begin{array}{c:c} 0 & -M \\ - - - & - - - \\ K & C \end{array}\right]$$

$$Q = M^{-1}\begin{bmatrix} 0 \\ - - - \\ F \end{bmatrix}$$

Then the first-order matrix equation of motion becomes $U = PZ + Q$, where U is the time derivative of Z.

If Z_j is defined to be the solution of the eigenvalue equation $PZ_j = \lambda_j Z_j$, the solution of the homogeneous differential equation of motion can be expressed as a modal series:

$$Z = \sum_j a_j(t) Z_j$$

where a_j, the complex amplitude of the j^{th} mode, is

$$a_j = a_j(0)\exp(\lambda_j t)$$

From the definition of P,

$$V_j = \lambda_j R_j$$

and

$$-M^{-1}(KR_j + CV_j) = \lambda_j V_j$$

Substituting for V_j yields the damped eigenvalue equation

$$\left(\lambda_j^2 M + \lambda_j C + K\right) R_j = 0$$

with 2n eigenvalues. If this equation is premultiplied by R_j^* (R_j conjugate transposed), it becomes

$$\lambda_j^2 m_j + \lambda_j c_j + k_j = 0$$

where the scalar modal mass, modal damping, and modal stiffness are defined as

$$m_j = \frac{R_j^* M R_j}{R_j^* R_j}$$

$$c_j = \frac{R_j^* C R_j}{R_j^* R_j}$$

$$k_j = \frac{R_j^* K R_j}{R_j^* R_j}$$

If the natural frequency is defined as

$$\omega_j = \sqrt{\frac{k_j}{m_j}}$$

and the damping ratio as

$$\zeta_j = \frac{c_j}{2\sqrt{m_j k_j}}$$

the eigenvalue is

$$\lambda_j = \omega_j \left(-\zeta_j \pm \sqrt{\zeta_j^2 - 1} \right)$$

The frequency is defined to be greater than zero. If the damping ratio is greater than zero, the mode is damped; if the damping ratio is less than zero, the mode is amplified.

The first-order matrix equation of motion defines U as a function of Z and t. The various numerical solutions of this set of differential equations are solutions of the following difference equations:

Forward Euler method:

$$Z(t + h) = Z(t) + hU[Z(t), t]$$

Backward Euler method:

$$Z(t + h) = Z(t) + hU[Z(t + h), t + h]$$

Centered Euler method:

$$Z(t + h) = Z(t) + \frac{h}{2}\left\{U[Z(t), t] + U[Z(t + h), t + h)]\right\}$$

Runge-Kutta method (fourth order):

$$Z(t+h) = Z(t) + \frac{g_1 + 2g_2 + 2g_3 + g_4}{6}$$

where

$$g_1 = hU[Z(t), t]$$

$$g_2 = hU\left[Z(t) + \frac{g_1}{2}, t + \frac{h}{2}\right]$$

$$g_3 = hU\left[Z(t) + \frac{g_2}{2}, t + \frac{h}{2}\right]$$

$$g_4 = hU[Z(t) + g_3, t + h]$$

Milne method:

$$Z(t+h) = Z(t-h) + \frac{h}{3}\left\{U[Z(t+h), t+h] + 4U[Z(t), t] + U[Z(t-h), t-h]\right\}$$

Adams method:

$$Z(t+h) = Z(t) + \frac{h}{24}\left\{9U[Z(t+h), t+h] + 19U[Z(t), t] - 5U[Z(t-h), t-h] + U[Z(t-2h), t-2h]\right\}$$

From the definition of U, these difference equations can be generalized as

$$Z(t+h) = \sum_{l} D_l Z(t - lh) + B_l Q(t - lh)$$

where l indicates the various terms used in the difference equations and D_l and B_l are matrices that are different functions of P and h for each difference equation. The exact solution to these difference equations is Z. The computation actually calculates

$$Z(t+h) + E(t+h) = \sum_{l} \left\{ D_l [Z(t - lh) + E(t - lh)] + B_l Q(t - lh) \right\} + G(t+h)$$

where E is a column matrix of the roundoff error and G is a column matrix of the rate of generation of the roundoff error. If the specific computer rounds the last significant figure (rather than truncating), the rate of generation of roundoff error, on the average, is zero. Subtracting the exact difference equation from the computational difference equation yields the following homogeneous difference equation for the roundoff error:

$$E(t+h) = \sum_{l} D_l E(t - lh)$$

The matrix D_l can be expressed as a polynomial function of hP

$$D_l = \sum_{k} d_{kl} (hP)^k$$

where d_{kl} is a different scalar constant for each integration technique and k indicates the various powers of the polynomial. Table I is a tabulation of d_{kl} for the various techniques. If E is expressed as a sum of eigenvectors (modal expansion), E becomes

$$E = \sum_{j} e_j(t) Z_j$$

where e_j is the complex amplitude of the roundoff error of the j^{th} mode. Substituting this into the difference equation for the roundoff error and noting that Z_j is a linear independent eigenvector of P result in the following equation for the amplitude of the roundoff error of the j^{th} mode:

$$e_j(t+h) = \sum_{l} \sum_{k} d_{kl} (h\lambda_j)^k e_j(t - lh)$$

This homogeneous difference equation has a solution of the form

$$e_j(t) = e_j(0) \exp(\mu_j t)$$

where μ_j is a constant. If w_j, the absolute error growth rate of the j^{th} mode, is defined as

$$w_j = \frac{e_j(t + h)}{e_j(t)}$$

w_j becomes

$$w_j = \exp(\mu_j h)$$

If s_j, a complex nondimensional time step for the j^{th} mode, is defined as $s_j = h\lambda_j$, this form of solution convertz the difference equation into an algebraic equation in terms of w_j and s_j:

$$w_j = \sum_l \sum_k d_{kl} s_j^k w_j^{-l}$$

If the relative error of the j^{th} mode is defined as

$$\epsilon_j(t) = \frac{e_j(t)}{a_j(t)}$$

the relative error growth rate of the j^{th} mode is

$$u_j = \frac{\epsilon_j(t + h)}{\epsilon_j(t)}$$

Substituting the values of a_j and ϵ_j and using the definitions of s_j and w_j yield

$$u_j = w_j \exp(-s)$$

The algebraic equation in terms of w_j and s_j is transformed into an equation in terms of u_j and s_j

$$u_j = \sum_l \sum_k d_{kl} s_j^k \exp[-(l + 1)s] u_j^{-l}$$

Both equations are polynomials in terms of either w_j or u_j with complex coefficients that are functions of s. These polynomials can be solved either analytically or numerically for w_j or u_j in terms of s. From the definition of s_j and the value of λ_j, s_j becomes

$$s_j = h\omega_j \left(-\zeta_j \pm \sqrt{\zeta_j^2 - 1} \right)$$

The largest absolute value of either w_j or u_j was used with a numerical contour plotting routine in terms of $h\omega_j$ and ζ_j. These contours are shown in figures 2 and 3.

DISCUSSION

A nonlinear finite element model of a rotor bearing system can be linearized at any instant of time. The stiffness and damping coefficients are assumed to be constant over a short time period at the instantaneous values. A damped critical-speed analysis can then be applied to obtain the instantaneous mode shapes, natural frequencies, and damping ratios. The analysis only applies for the time period for which the linearization is valid. For other time periods the analysis must be repeated with "new" stiffness and damping coefficients.

A numerical stability analysis was applied to the linearized finite element model of the rotor bearing system to determine the stability limits of the forward, backward, and centered Euler; the fourth-order Runge-Kutta; the Milne; and the Adams methods of numerical integration of a set of differential equations. The stability analysis obtained the error growth rate (the ratio of the amplitudes of the errors for two succeeding time steps) as a function of a nondimensional time step (the product of the time step and the natural frequency) and of the damping ratio for each mode. Figure 2 shows contour lines of the absolute error growth rate and figure 3 shows contour lines of the relative error growth rate. The relative error growth rate was normalized by the transient solution of the perfectly balanced rotor.

In the finite element model of the rotor bearing system, the higher frequency modes are not physically meaningful but are inherently present. The accuracy of the calculations in these modes is not important. It is only important that, for a nonnegative damping ratio, the absolute error be bounded (i.e., the absolute error growth rate must be less than or equal to 1). If the absolute error growth rate is greater than 1, eventually the numbers used in the calculation will become too large for the computer and a computer overflow will occur. This is not too serious for the negative damping ratio case (amplification) since the rotor vibrations will be unbounded and the actual displacements will eventually cause the computer to overflow.

The lower frequency modes are the only modes that are physically meaningful. The accuracy of the calculations in these modes is important. These modes must have a relative error that is bounded (i.e., the relative error growth rate must be less than or equal to 1). If the relative error growth rate is greater than 1, eventually the numbers used in the calculation will become meaningless. The relative error growth rate shown in figure 3 is for a rotor that is in perfect balance. This will yield the smallest vibrations and therefore the largest relative error. A rotor that is out of balance will have larger vibrations and therefore a smaller relative error. If the integration technique is to be universal (i.e., applying to all transients), relative error growth rate as shown in figure 3 must be less than or equal to 1.

In general, increasing the damping ratio while keeping the product of the time step and the natural frequency fixed causes the relative error to grow. This is also true for the absolute error, with the exception of the centered and backward Euler methods. For damping ratios between -0.5 and 0.7 for the forward Euler method, between 0.5 and ∞ for the backward Euler method, between 0 and 0.8 for the centered Euler method, between 1.7 and ∞ for the Runge-Kutta method, between 0 and ∞ for the Milne method, and between $-\infty$ and -1.7 for the Adams method, the relative error growth rate is greater than 1.

EXAMPLE

As an example, consider a uniform shaft on rigid bearings. The damping ratio is zero for all modes; thus, the absolute and relative error growth rates are equal. The natural frequency for a continuous model of this system is

$$\omega_j = j^2 \omega_1$$

As an order-of-magnitude approximation, the natural frequencies of the finite element model are approximately equal to the natural frequencies of the continuous model. If there are n elements in the model, the highest natural frequency is

$$\omega_n = n^2 \omega_1$$

The n^{th} mode (not usually desired from the calculations but inherently present) determines the threshold of the numerical instability. From figure 3 the forward Euler method is always unstable. The backward and centered Euler methods are never unstable. The Runge-Kutta method is unstable for a time step greater than approximately

$$h > \frac{3}{n^2 \omega_1}$$

The Milne method is unstable for a time step greater than approximately

$$h > \frac{2}{n^2 \omega_1}$$

The Adams method is unstable for a time step greater than approximately

$$h > \frac{1}{n^2 \omega_1}$$

If the shaft is running at approximately the first natural frequency and if there are 10 elements, the calculation must be done at least every 1.5, 1, and 0.5 degrees of shaft rotation for the Runge-Kutta, Milne, and Adams methods, respectively. To calculate the shaft motion to several revolutions would require thousands of time steps.

If these time-step limits are exceeded by a factor of 2, the error growth rates are approximately 10, 3, and 1.5 for the Runge-Kutta, Milne, and Adams methods, respectively. This means that a relative error of 10^{-8} will grow to a value of 1 in less than 8, 17, and 46 time steps for each of these methods, respectively.

The preceding example illustrates the fact that great care should be taken to minimize the number of elements in the finite element modal, thus maximizing the time step. This is important not only to save calculation time but also because the time step has a lower limit dictated by the roundoff error.

CONCLUSIONS

The numerical stability of the Euler, Runge-Kutta, Milne, and Adams integration techniques as applied to transient rotor dynamics was analyzed. The following conclusions were drawn:

1. The highest frequency mode possible in the finite element modal usually determines the threshold of numerical stability. Therefore, the number of mass elements in the finite element model should be minimized.

2. Increasing the damping ratio of a mode can cause it to become numerically unstable.

3. Numerical stability for the Runge-Kutta, Milne, and Adams methods for a uniform shaft on rigid bearings with 10 mass elements (operating approximately at the first critical speed) is to make a calculation at less than 1 degree of shaft rotation.

In general, the product of the time step and the natural frequency, for all modes possible in the finite element model of the rotor bearing system, must be less than the stability threshold.

Lewis Research Center,
National Aeronautics and Space Administration,
and
U. S. Army Air Mobility R&D Laboratory,
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APPENDIX - SYMBOLS

A time derivative of V

a complex amplitude

B column matrix functions of P and h

C square matrix describing damping in finite element model

c modal damping

D column matrix functions of P and h

d scalar coefficient of polynomial of hP

E column matrix of roundoff error

e amplitude of absolute error

F column matrix describing forces in finite element model

G column matrix of rate of generation of roundoff error

g parameters used in Runge-Kutta method

h time step

K square matrix describing stiffnesses in finite element model

k modal stiffness

M diagonal matrix describing masses in finite element model

m modal mass

n number of mass elements in finite element model

P partitioned square matrix used in homogeneous first-order matrix equation of motion

Q partitioned column matrix describing forcing function in first-order matrix equation of motion

R column matrix describing displacements of shaft centerline in finite element model

r complex elements of R

s complex nondimensional time step

t time

U time derivative of Z

u relative error growth rate

V time derivative of R

w absolute error growth rate

Z partitioned column matrix describing dependent variables in first-order matrix equation of motion

ϵ amplitude of relative error

ζ damping ratio

λ complex (damped) eigenvalue

μ constant used in solution of finite difference equations

ω natural frequency

Subscripts:

j any mode between 1 and n

k power of polynomial of hP

l various terms of difference equation

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TABLE I. - TABULATION OF d_{kl} FOR VARIOUS TECHNIQUES

(a) Forward Euler

Term of difference equation, l	Power of polynomial of hP, k	
	0	1
0	1	1

(b) Backward Euler

Term of difference equation, l	Power of polynomial of hP, k	
	0	1
-1	0	1
0	1	0

(c) Centered Euler

Term of difference equation, l	Power of polynomial of hP, k	
	0	1
-1	0	1/2
0	1	1/2

(d) Runge-Kutta

Term of difference equation, l	Power of polynomial of hP, k				
	0	1	2	3	4
0	1	1	1/2	1/6	1/24

(e) Milne

Term of difference equation, l	Power of polynomial of hP, k	
	0	1
-1	0	1/3
0	0	4/3
1	1	1/3

(f) Adams

Term of difference equation, l	Power of polynomial of hP, k	
	0	1
-1	0	9/24
0	1	12/24
1	0	-5/24
2	0	1/24

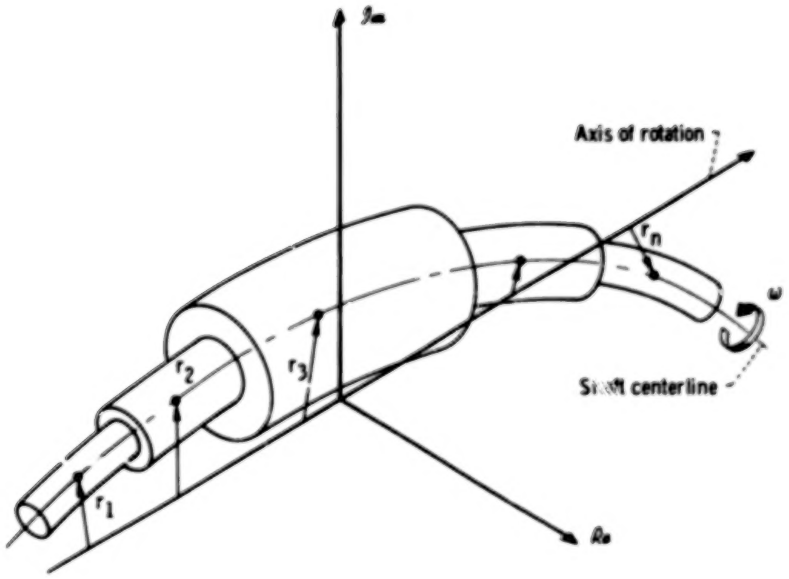

Figure 1. - Model of shaft showing complex number representation of radial displacements (distance between shaft centerline and axis of rotation). Real and imaginary axes fixed in space.

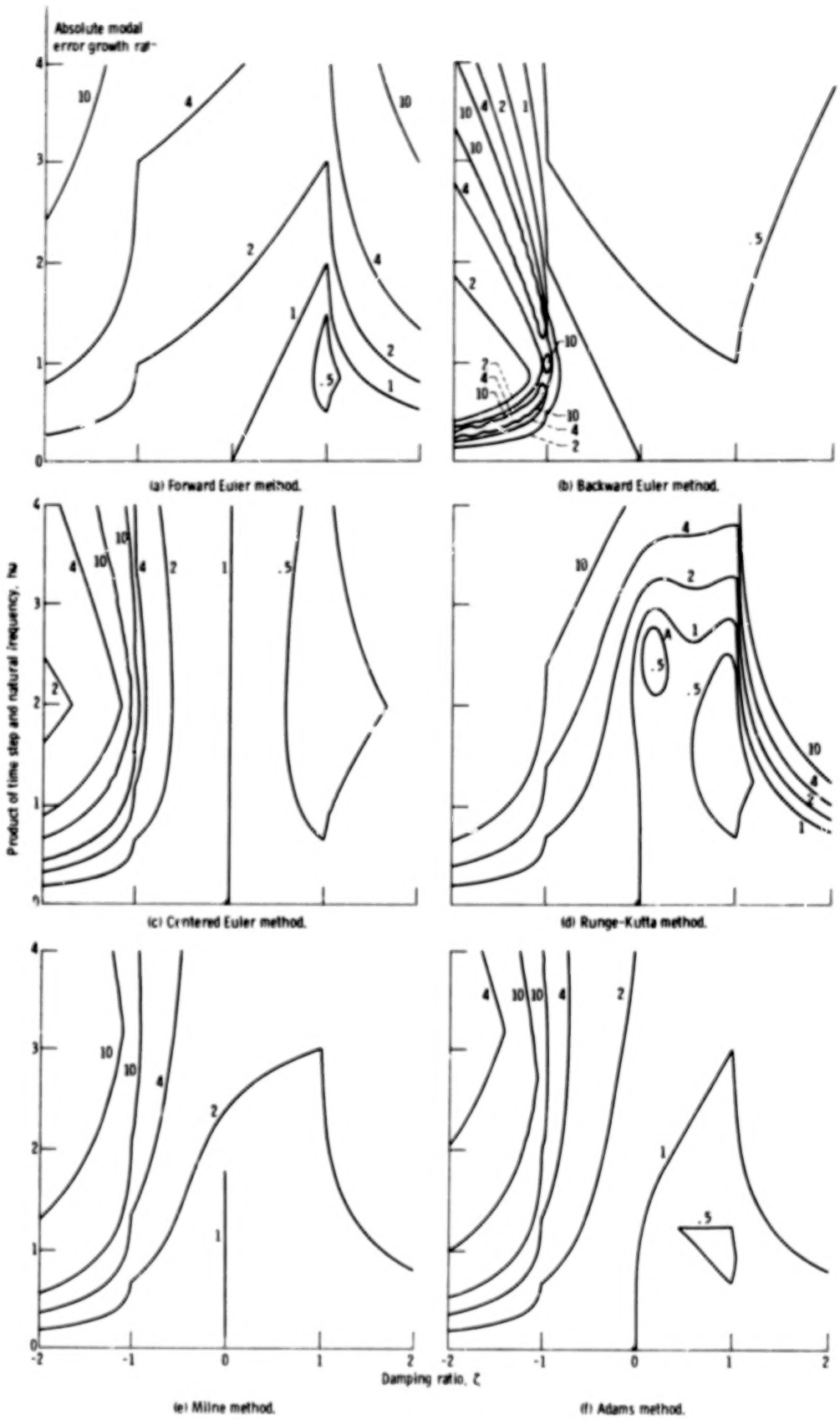
Absolute modal error growth rate
(a) Forward Euler method.
(b) Backward Euler method.
(c) Centered Euler method.
(d) Runge-Kutta method.
(e) Milne method.
(f) Adams method.
Product of time step and natural frequency, hω
Damping ratio, ζ

Figure 2. - Contours of absolute modal error growth rate.

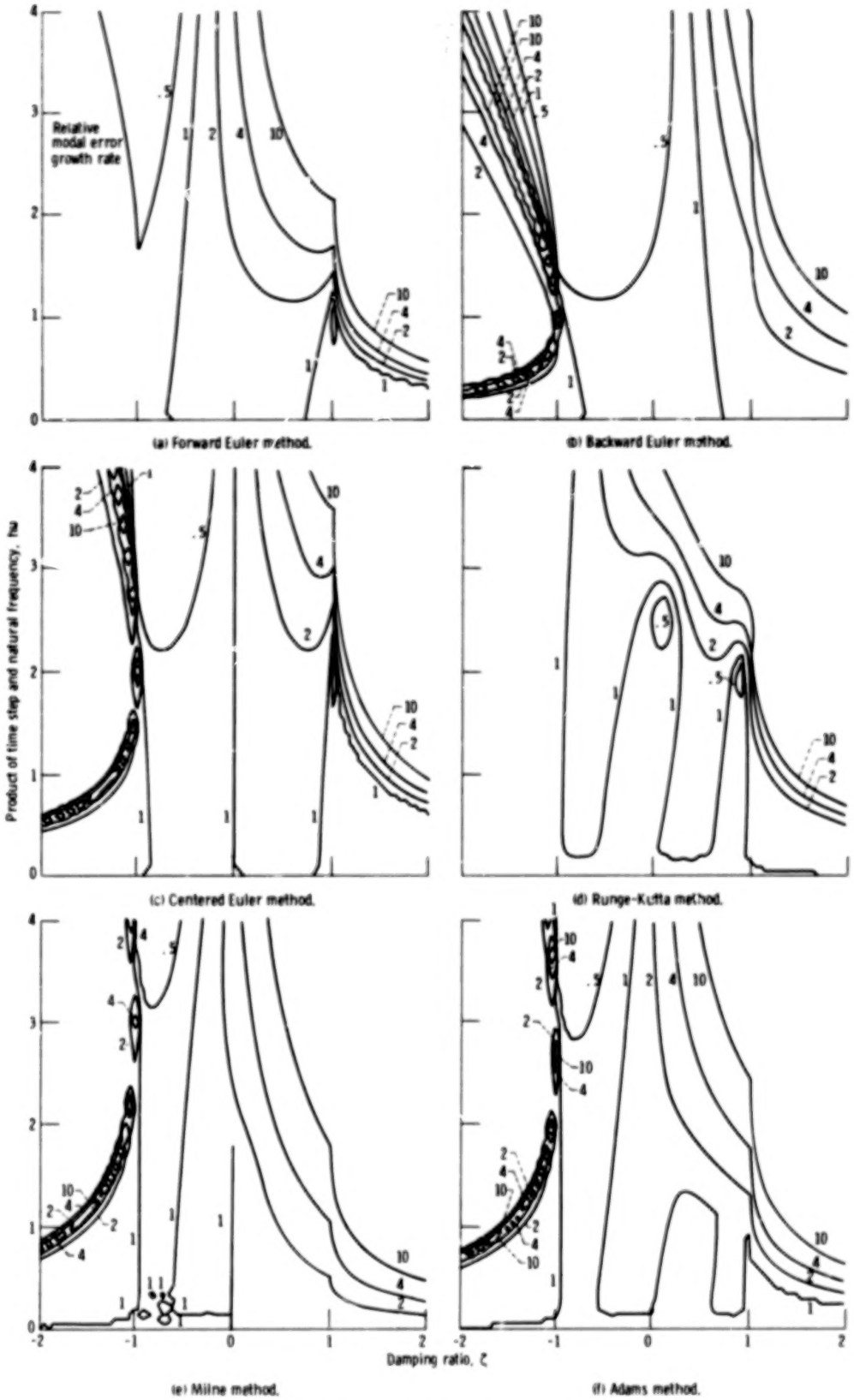
Relative modal error growth rate
Product of time step and natural frequency, hω
Damping ratio, ζ
(a) Forward Euler method.
(b) Backward Euler method.
(c) Centered Euler method.
(d) Runge-Kutta method.
(e) Milne method.
(f) Adams method.

Figure 3. - Contours of relative modal error growth rate.

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16. Abstract

A finite element model of a rotor bearing system was analyzed to determine the stability limits of the forward, backward, and centered Euler; Runge-Kutta; Milne; and Adams numerical integration techniques. The analysis concludes that the highest frequency mode determines the maximum time step for a stable solution. Thus, the number of mass elements should be minimized. Increasing the damping can sometimes cause numerical instability. For a uniform shaft, with 10 mass elements, operating at approximately the first critical speed, the maximum time step for the Runge-Kutta, Milne, and Adams methods is that which corresponds to approximately 1 degree of shaft movement. This is independent of rotor dimensions.

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